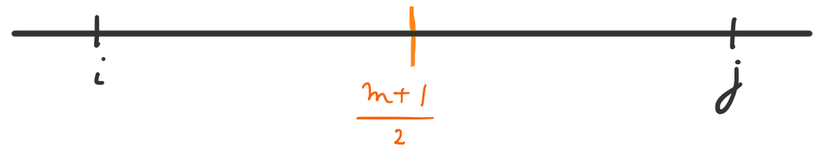
Exercise 9.6

Tuesday, 17 October 2023 15:44

$$X_i = \begin{cases} 1 & \text{if } i^{th}\text{-largest element is a pivot} \\ 0 & \text{otherwise} \end{cases}$$



$$X = \sum_{i \in [m]} X_i \quad = \text{number of passes} \quad = X^+ + X^- + 1$$

(for the median)

$$\Pr[X_i = 1] = \frac{1}{\left|\frac{m+1}{2} - i\right|}$$

distance from median

$$X^- = \sum_{i=1}^{\frac{m+1}{2}-1} X_i$$

$$X^+ = \sum_{i=\frac{m+1}{2}+1}^{m} \frac{1}{i - \left(\frac{m+1}{2}\right)}$$

$$E[X] = E\left[\sum_{i=1}^{\frac{m+1}{2}-1} X_i\right] = \sum_{i=1}^{\frac{m+1}{2}-1} E[X_i]$$

$$= \sum_{i=1}^{\frac{m+1}{2}-1} \Pr[X_i = 1]$$

$$= \sum_{i=1}^{\frac{m+1}{2}-1} \frac{1}{\frac{m+1}{2} - i}$$

$$= \sum_{i=1}^{\frac{m+1}{2}-1} \frac{1}{i}$$

$$= \Theta\left(\log\left(\frac{m+1}{2} - 1\right)\right)$$

$$= \Theta(\log(m))$$