The simplified recurrence becomes

$$T_{IO}(n) \leq \begin{cases} 2T_{IO}\left(\frac{n}{2}\right) + O\left(\frac{n}{B}\right) & \text{if } n > \frac{M}{2} \\ O\left(\frac{M}{B}\right) & \text{otherwise} \end{cases}$$

$\frac{n}{B}$ sum $\frac{n}{B}$

$\frac{n}{2B}$ $\frac{n}{2B}$ $\frac{n}{B}$

$\frac{n}{4B}$ $\frac{n}{4B}$ $\frac{n}{4B}$ $\frac{n}{4B}$ $\frac{n}{B}$

problem size $= \frac{n}{B \cdot 2^i}$ Recursion stops when problem size $\leq \frac{M}{2}$

← height of tree

number of base cases $= 2^h$

$$\frac{n}{B \cdot 2^h} = \frac{M}{2} \longrightarrow 2^h = \frac{2n}{MB}$$
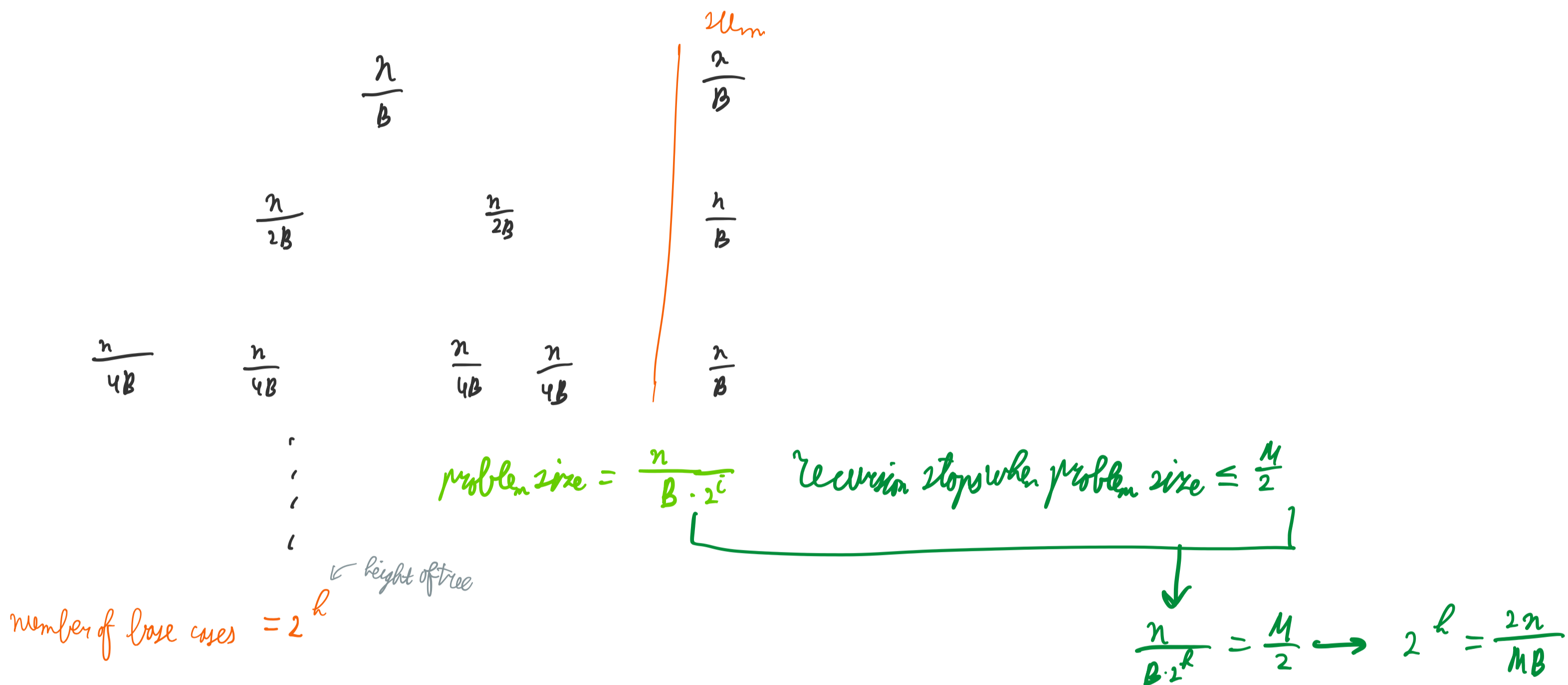
recurrence overhead = height of tree · overhead per level

total $\# IOs$ = $\# $ base cases · $\#IOs$ per base case + recurrence overhead

$$= 2^h \cdot O\left(\frac{M}{B}\right) + \log_2\left(\frac{2n}{MB}\right)\frac{n}{B}$$

$$= O\left(\frac{2n}{MB}\frac{M}{B} + \log_2\left(\frac{2n}{MB}\right)\cdot\frac{n}{B}\right)$$

$$= O\left(\frac{n}{B^2} + \log_2\left(\frac{n}{MB}\right)\cdot\frac{n}{B}\right)$$

$$= O\left(\frac{n}{B} + \log_2\left(\frac{n}{MB}\right)\cdot\frac{n}{B}\right) \quad \text{for } B \geq 1$$

$$= O\left(\log_2\left(\frac{n}{MB}\right)\cdot\frac{n}{B}\right)$$

$$= O\left(\frac{n}{B}\log_2\left(\frac{n}{M}\right)\right) \quad \text{we drop the } \frac{1}{B} \text{ in the logarithm}$$